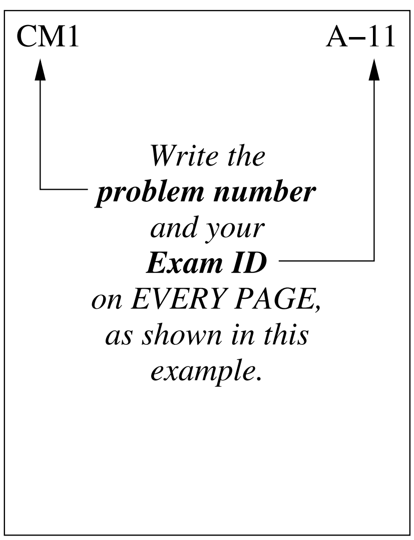
Department of Physics
Montana State University

Qualifying Exam
January, 2022

Day 1
Classical Mechanics



- Show your work.
- Write your solutions on the blank paper that is provided.
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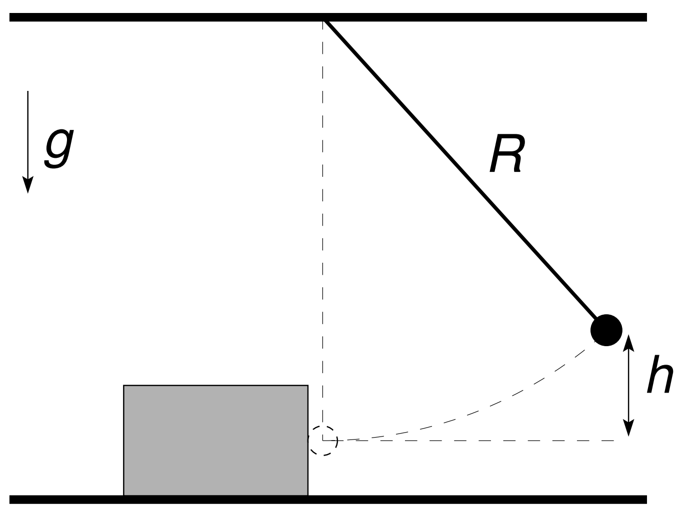
(CM1) A pendulum consists of a massless rigid rod of length R with a weight of mass m attached to its end. There is a block of mass $2m$ resting on a table, whose end is exactly below the suspension point of the pendulum. There is no friction between block and table. The bob is released from a height h above the lowest position of the pendulum, and it collides elastically with the block. After the collision:

a) To what height does the bob rise, and does it rebound or continue in the same direction?

b) Describe the result if the bob had mass $2m$ and the block mass m.



(CM2) A particle with mass m moves in the xy-plane in an ellipse of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{1}$$

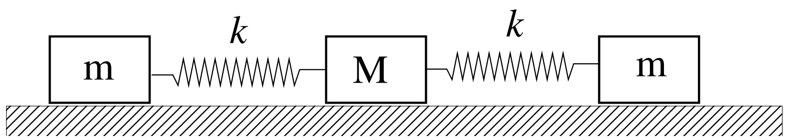
and in two seconds the particle orbits this ellipse three times.

a) Find an expression for the particle's trajectory $\mathbf{r}(t) = (x(t), y(t), z(t))$, taking $x(t) = a \cos \omega t$.

b) Find the force acting on the particle.

c) Calculate the angular momentum of the particle. Why is its angular momentum constant in magnitude and direction?

(CM3) Three masses, m, M, m are connected by springs with constants k, as shown in the figure. The blocks are on a frictionless table; they all can only move along one line. Find the normal vibrational modes of the system.



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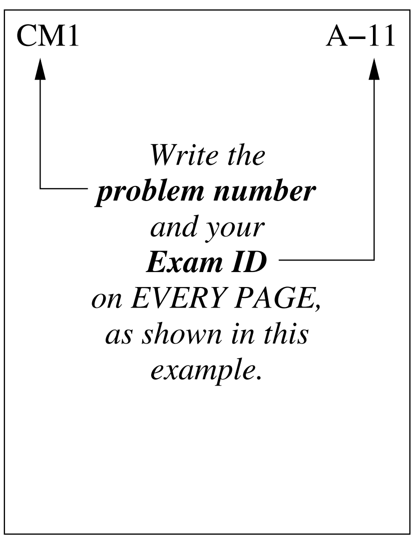
Day 2
Quantum Mechanics



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(QM1) An electron is confined to a 1D infinite square well with infinite potential barriers at $x = 0$ and $x = a$.

The infinite square well is perturbed with the following repulsive core:

$$\hat{H}' = \begin{cases} V_0 & a/4 \leq x \leq 3a/4 \\ 0 & \text{otherwise} \end{cases}$$

Here, $V_0 = |E_g|/10$ where E_g is the ground state energy of the unperturbed well.

(a) To first-order in V_0, estimate the ground state energy of the electron in the well with the repulsive core. Does the ground state energy increase or decrease with respect to the unperturbed well?

(b) To first-order in V_0, estimate the ground state wavefunction of the electron in the well with the repulsive core. Describe how the ground state wavefunction is modified due to the repulsive core. **Note:** only consider contributions from the first three lowest-energy unperturbed states.

As always with the infinite square well, the angle addition trig identities are useful for the calculations:

$$\sin(a)\sin(b) = \frac{\cos(a-b) - \cos(a+b)}{2}$$
$$\cos(a)\cos(b) = \frac{\cos(a+b) + \cos(a-b)}{2}$$

(QM2) In the hydrogen atom with only the central-potential coulombic interaction, each energy level E_n^0 is degenerate, with $2n^2$ electronic states all having the same energy. These states are $|n, l, m, m_s\rangle$, where $n = 1, 2, 3\ldots$ is the principal quantum number, $l = 0, 1, \ldots n-1$ is the orbital angular momentum quantum number, $m = -l \ldots l$ is projection of orbital angular momentum $\mathbf{L}$ on z-axis, and $m_s = \pm 1/2$ is the projection of electron spin $\mathbf{S}$ on z-axis.

Relativistic effects result in spin-orbit interaction V_{SO} between the orbital $\mathbf{L}$ and spin $\mathbf{S}$ angular momenta of the electron, that partially lifts the degeneracies (i.e., it splits the degenerate states into states with different energies). Taking

$$V_{SO} = a\,\mathbf{L}\cdot\mathbf{S} \qquad \text{with } a\text{=const},$$

introduce appropriate new quantum numbers and determine the number of split levels, their energies and degeneracies.

(QM3) A particle of mass m in an infinite square well, $-d/2 < x < d/2$, is initially in the state

$$\psi_0(x) = A\cos\left(\frac{\pi x}{d}\right) + A\sin\left(\frac{2\pi x}{d}\right), \quad t = 0.$$

Find all times $t \geq 0$ when the expectation value of the particle's position, $\langle x(t)\rangle$, is 0.

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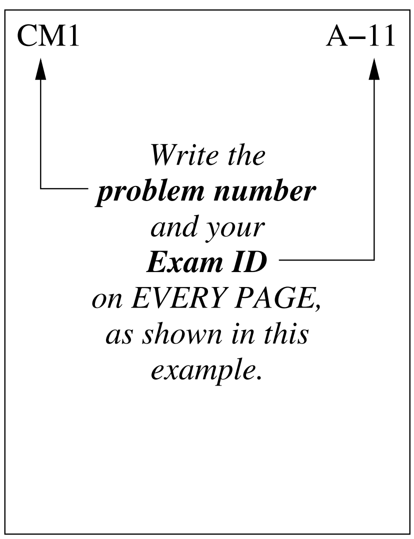
Day 3
Electricity and Magnetism



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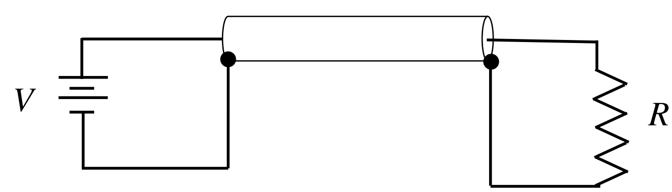
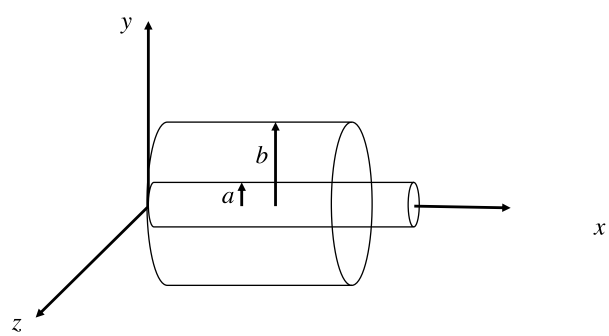
(EM1) A constant voltage, V, is connected to a simple resistor R by a length of coaxial cable ("coax") as shown diagram (a). *Assume that the coax is made of a perfect conductor.* The radius of the coax's central conductor is a and the inner radius of the outer conductor is b (see diagram (b)). Find the Poynting vector $\vec{S}$ in the coax's dielectric and relate it to the instantaneous power P in the resistor.

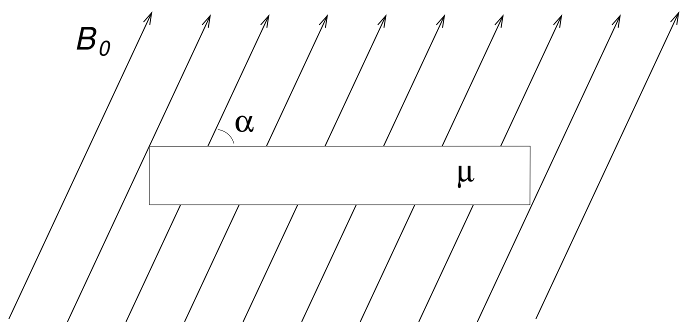
(a)



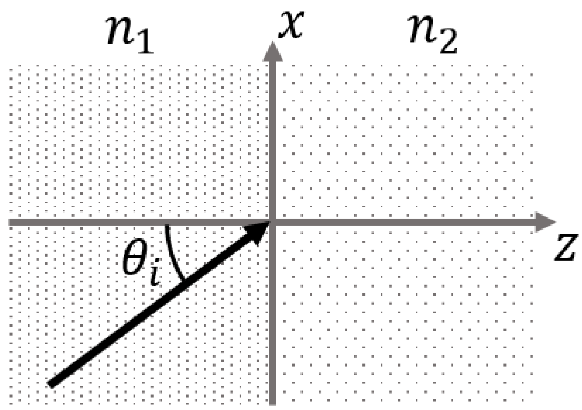
(b)



(EM2) A disk of linear magnetic material with permeability μ is in vacuum and is placed into a uniform magnetic field B_0, that makes angle α with its surface. Ignoring the edge effects, determine the magnitude and direction of the magnetization in the disk. (In the figure we view the disk from its side, and the thickness of the disk is much smaller than the other dimensions.)



(EM3) A plane-wave with angular frequency ω is incident on the interface at $z = 0$ between two dielectric media with indices of refraction n_1 and n_2 where $n_1 > n_2$ as shown below. The angle of incidence is θ_i. You may, if you wish, assume that incident wave has its electric field polarized normal to the plane-of-incidence (i.e., in the $\hat{y}$ direction).



(a) For the transmitted wave beyond the interface, determine its angular frequency and derive Snell's Law that describes its angle of refraction using the appropriate boundary conditions for the electric and magnetic fields.

(b) On a schematic similar to what is shown above and includes the interface, qualitatively compare the angle of incidence to the angle of refraction. Which is larger?

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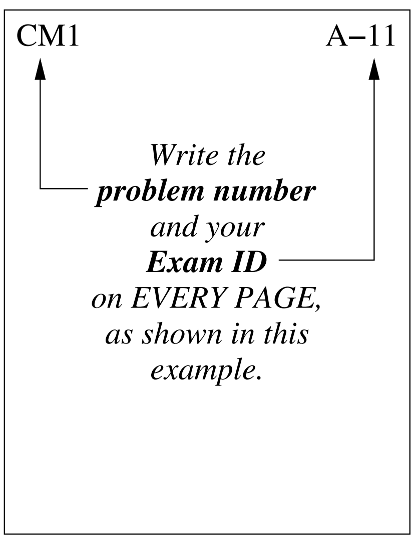
Day 4
Statistical and Thermal Physics



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(ST1) Consider the air in a valley as an ideal gas.

(a) Provide a brief argument why the pressure P as a function of height z above the valley floor behaves as $P(z) = P(0)\, e^{-mgz/k_B T}$, where m is the average mass of the air molecules and T is assumed to be approximately constant.

(b) A 'parcel' of dry air at $z = 0$ is gently pushed by the wind up a mountain slope. During this process, imagine that the 'parcel' expands adiabatically, maintaining pressure equilibrium with its surroundings. Find an expression for the change in temperature of the 'parcel' with height z, making use of the relation in part (a).

(c) Obtain an order of magnitude estimate for the temperature change of the 'parcel' for each 100 meters of ascent.

Useful information: The acceleration of gravity is $g = 9.8$ m/s^2, the average mass of an air molecule is 5×10^{-26} kg, and $\mathrm{k_B} = 1.38 \times 10^{-23}$ J/K. Treat the air molecules as diatomic, with heat capacity per molecule $c_v = 5/2$. The adiabatic constant $\gamma = c_p/c_v = 1 + 1/c_v = 7/5$.

(ST2) Within an evacuated enclosure at absolute temperature T the radiant energy per unit volume ($u = E/V$) within the enclosure is given by $u = u(T)$. From electromagnetic theory, the pressure that the radiant energy exerts on the container walls is $P = u/3$. Considering E as a function of T and V, use the combined first- and second-laws, *i.e.*, $dS = (dE + PdV)/T$, to show that $u =$ (constant) T^4.

(ST3) The three lowest energy levels for a molecule are $E_0 = 0$, $E_1 = \epsilon$, and $E_2 = 10\epsilon$.

(a) Find the contribution of these three levels to the specific heat per mole, C_v, of a gas composed of these molecules.

(b) Sketch C_v as a function of temperature (T) paying particular attention to the asymptotic behaviors as $T \to 0$ and $T \to \infty$.