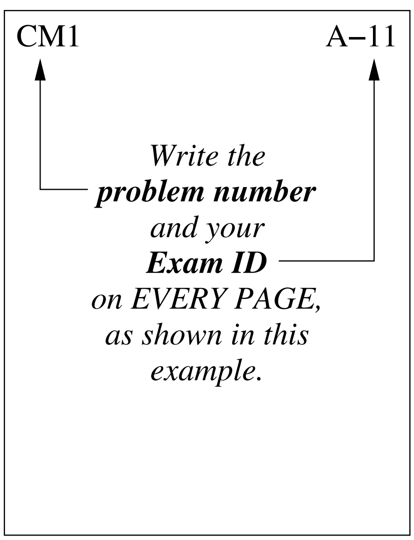
Department of Physics
Montana State University

Qualifying Exam
August, 2022

Day 1
Classical Mechanics



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- Write your solutions on the blank paper that is provided.
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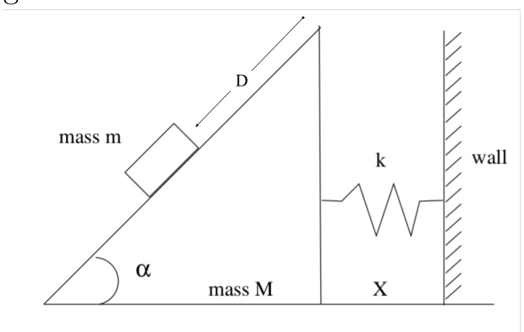
(CM1) A block of mass m slides with no friction down a ramp of mass M and height L under the force of gravity. The ramp is attached to the wall by a spring with spring constant k.



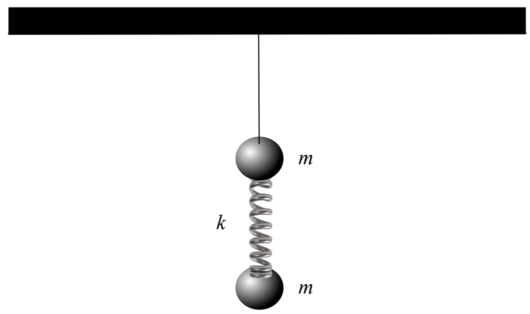
(a) Write the Lagrangian of the system in terms of X, the distance of the ramp from the wall, and of D, the distance of the object from the top of the ramp.

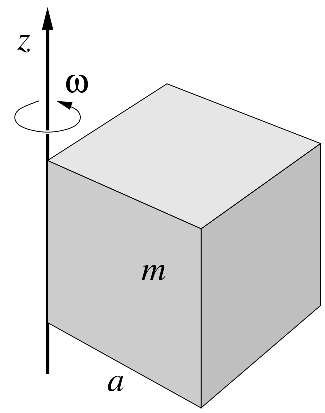
(b) Write the coupled equations of motion for these generalized coordinates.

(c) Suppose that α is small, indeed take it to zero for the purpose of this calculation. For $\alpha = 0$, find the normal frequencies and the normal modes.

(d) Describe the motion resulting from the two normal modes.

(CM2) Two small balls of mass m are connected by a massless spring with a spring constant k, and the setup is attached to the roof by a massless string as shown in the figure. If the string is cut and the setup drops from rest, find the motion of the two balls.





(CM3) A cube of mass m has constant density and edge length a. The cube rotates about a fixed axis (z) on one edge, at fixed angular frequency ω.

(a) Find the cube's angular momentum about the z-axis, and its kinetic energy.

(b) Calculate the (linear, instantaneous) velocity of the center of mass. What is the direction and magnitude of the force on the cube?

(c) The cube is released from its axis, so that there are no forces acting on it. Describe its subsequent angular and linear velocities. Compare the kinetic energy immediately before and after release. Do the same for angular momentum.

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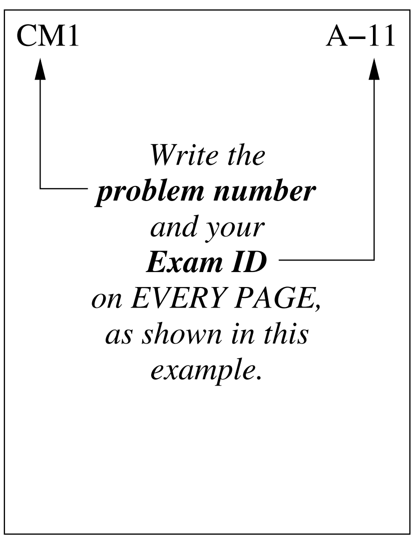
Day 2
Quantum Mechanics



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(QM1) A particle with spin $\hbar/2$, and magnetic moment $\boldsymbol{\mu} = \gamma \mathbf{S}$, where $\mathbf{S}$ is the spin operator and $\gamma > 0$ is the gyromagnetic ratio, sits in a uniform magnetic field that suddenly switches direction at $t = 0$:

$$\mathbf{B}(t) = \begin{cases} B_0 \hat{x} \,, & \text{for} \quad t < 0 \\ B_0 \hat{z} \,, & \text{for} \quad t > 0 \end{cases}$$

(a) Find the spinor wave function and the expectation value of the spin for $t < 0$ given that it is in the ground state of the system.

(b) Find the expectation value of the spin $\langle \mathbf{S}(t) \rangle$ for $t > 0$.

(c) Make a plot of the spin components $\langle S_i(t) \rangle$ as function of time.

Reminder: the spin-1/2 matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

(QM2) A particle with mass m is confined to a 2D infinite square well with infinite potential barriers at $x = -a$, $x = a$, $y = -a$, and $y = a$.

The system experiences a weak potential that has the following form:

$$\hat{H}_1 = \beta x y$$

Here, β is a constant.

(a) Determine the energies, states, and degeneracies of the ground and first excited state of the unperturbed system.

(b) Treating the potential as a perturbation, determine the energies and states of the first two *excited* states of the perturbed system to lowest order in $\hat{H}_1$.

Note: The following integral will be useful:

$$\int_{-\pi/2}^{\pi/2} u \cos u \sin 2u \; du = 8/9$$

(QM3) Consider a quantum particle with mass m in a 1D harmonic potential of the form $V(x) = \frac{1}{2}\omega^2 x^2$.

(a) Determine how the product of the squares of the uncertainty of position and momentum ($\sigma_x^2 \sigma_p^2$) depend on the principle quantum number of the system, n. Find the algebraic dependence and make a plot for all quantum numbers up to $n = 2$.

(b) Of all possible energy eigenstates, which (if any) are states of minimum uncertainty (i.e. which energy eigenstates are at the limit established by the Heisenberg uncertainty principle)?

The following relationships using the ladder operators may be useful

$$\begin{aligned}
\hat{a} &= \sqrt{\frac{m\omega}{2\hbar}}(\hat{x} + \frac{i}{m\omega}\hat{p}) \\
\hat{a}^\dagger &= \sqrt{\frac{m\omega}{2\hbar}}(\hat{x} - \frac{i}{m\omega}\hat{p}) \\
\hat{a}\,|n\rangle &= \sqrt{n}\,|n-1\rangle \\
\hat{a^\dagger}\,|n\rangle &= \sqrt{n+1}\,|n+1\rangle
\end{aligned}$$

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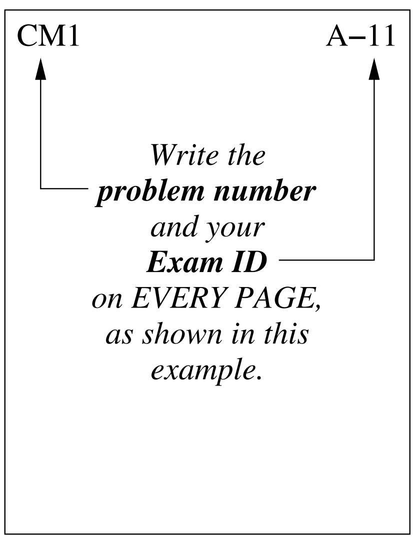
Day 3
Electricity and Magnetism



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Information

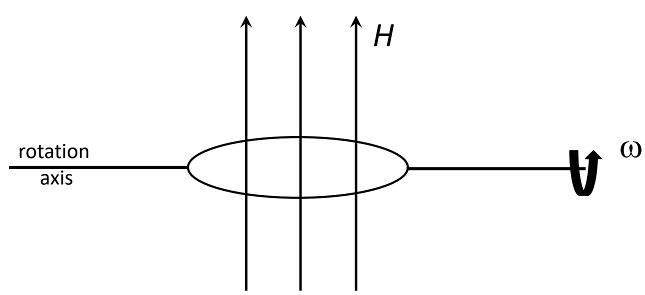
Useful vector identity

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A}$$

Table 1: PHYSICAL CONSTANTS

SYMBOL	NAME	VALUE	UNITS
c	speed of light in vacuum	299792458	m s^{-1}
G	gravitational constant	$6.67408 \cdot 10^{-11}$	N m^2 kg^{-2}
g	standard gravity	9.80665	m s^{-2}
h	Planck constant	$6.62607015 \cdot 10^{-34}$	J s
		$4.13566770 \cdot 10^{-15}$	eV s
$\hbar = h/2\pi$	reduced Planck constant	$1.05457182 \cdot 10^{-34}$	J s
		$6.58211957 \cdot 10^{-16}$	eV s
e	elementary charge	$1.602176634 \cdot 10^{-19}$	C
ε_0	electric constant	$8.854 \cdot 10^{-12}$	C^2 N^{-1} m^{-2}
μ_0	magnetic constant	$4\pi \cdot 10^{-7}$	T m A^{-1}
N_A	Avogadro's constant	$6.02214076 \cdot 10^{23}$	mol^{-1}
k_B	Boltzmann's constant	$1.380649 \cdot 10^{-23}$	J K^{-1}
$R = N_A k_B$	gas constant	8.314462618	J mol^{-1} K^{-1}
$\sigma = \frac{\pi^2 k_B^4}{60\hbar^3 c^2}$	Stefan-Boltzmann constant	$5.670367 \cdot 10^{-8}$	W m^{-2} K^{-4}
m_e	electron mass	$9.109 \cdot 10^{-31}$	kg
		0.5109	MeV
m_p	proton mass	$1.672 \cdot 10^{-27}$	kg
		938.2	MeV
m_n	neutron mass	$1.674 \cdot 10^{-27}$	kg
		939.5	MeV

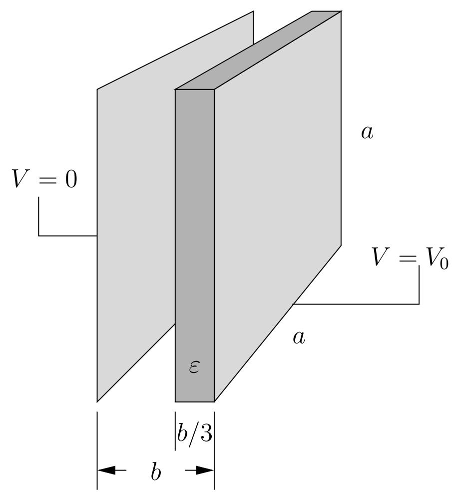
(EM1) A thin ring of copper with electrical conductivity σ, density ρ, mass m, and radius r rotates about an axis perpendicular to a uniform magnetic field H with initial angular frequency ω_0. Under the assumptions that the energy goes into Joule heating and ω_0 is rapid enough that the ring can complete many full rotations, find an expression for the time it takes for the angular frequency to decrease to ω_0/e (note $\ln(e) = 1$) in terms of the given quantities. Ignore the ring's self inductance.



(EM2) Visible light is incident normally on an aluminum (Al) plate. Describe how the electric (and magnetic) fields behave inside a thick plate, and use this to estimate how thick should the plate be to reduce the transmitted light *power* to 10^{-6} of the incident power. (You can ignore all reflections inside the plate to estimate transmission.)

Take the visible light frequency to be $\nu = 5 \cdot 10^{14}$ Hz with wavelength $\lambda \sim 700$ nm. The conductivity of Al is $\sigma = 3.5 \cdot 10^{7}\ \Omega^{-1}\,\mathrm{m}^{-1}$ (SI units) $= 3 \cdot 10^{17}\ \mathrm{s}^{-1}$ (Gaussian, cgs units).

(EM3) Two thin square, parallel metal plates, $a \times a$, are separated by a gap $b \ll a$. The gap is 1/3 filled by dielectric, permittivity ε, attached to one plate as shown. The plate adjacent to the dielectric is at potential V_0, and the other is grounded. Find the electric field between the plates, ignoring edge effects. Also find the bound and free charge densities.



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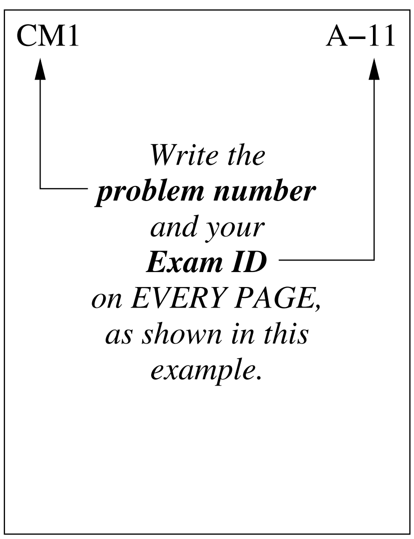
Day 4
Statistical and Thermal Physics



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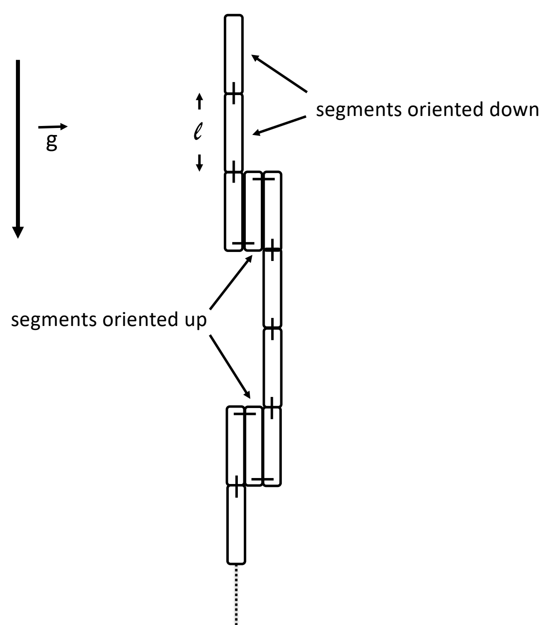
(ST1) The Sun has a surface temperature of $T_S = 5500$ K and its radius is $R_S = 7.0\times10^8$ m. The Earth has a radius of $R_E = 6.4\ \times10^6$ m and it is a mean distance $d = 1.5\times10^{11}$ m from the sun. Assume that both the Sun and the Earth absorb all electromagnetic radiation that is incident upon them, and that the Earth's temperature T_E is in a steady state, so that it does not change with time. Find an expression for T_E and provide an approximate numerical value.

(ST2) A rubber band can be modeled as a chain of N segments of identical length ℓ joined end to end. Imagine that a weight W hangs from the very end of our rubber band while the other end is firmly fastened to a peg anchored to the wall. The temperature is T. Each segment can be in one of two states, down or up, relative to the vertical. The figure illustrates a short length of the rubber band with segments oriented up and down. Determine the average length $\bar{L}$ of the rubber band as a function of W and T. Show that your model agrees with your physical expectations for $\bar{L}$ at low and high T.



(ST3) A sealed can of volume V_1 containing air at room temperature ($T_1 = 300\,\mathrm{K}$) is run over and flattened by a large, fast moving truck. Amazingly, the can does not leak! At the moment the truck is on top of the can, its volume is $V_2 \approx V_1/32$. Estimate the temperature, T_2, of the air in the can at that moment. Since air is an ideal gas and is mostly diatomic, you may assume $C_P = 7R/2$ and $C_V = 5R/2$.