Condensed Matter - HW 12 :: Ginzburg-Landau theory

PHSX 545

Problem 1 Superconductor in magnetic field

Starting from the Ginzburg-Landau functional

$$F[\psi] = \int dV \left\{ K \left| \left(\boldsymbol{\nabla} - i\frac{2e}{\hbar c}\mathbf{A} \right) \psi \right|^2 + a(T-T_c)|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{\mathbf{B}^2}{8\pi} \right\}$$

(a) Find the condensation energy $\Delta F(T)$ of uniform state in zero field, and determine the thermodynamic critical field defined by $\Delta F(T) = -VH_c^2/8\pi$.

(b) Define the coherence length $\xi(T)$ and penetration length $\lambda(T)$, in terms of parameters K, a, β and fundamental constants.

(c) Derive the linearized equation for ψ in magnetic field, and determine $H_{c2}(T)$ by solving the eigenvalue problem, i.e. find maximum field where first non-zero solution for ψ is possible. (Take vector potential in the form $\mathbf{A} = (0, Hx, 0)$ and recall solution of Schrödinger equation in uniform magnetic field.)

(d) From the above determine the critical value of parameter $\varkappa = \lambda/\xi$ when $H_{c2}(T)$ exceeds $H_c(T)$.

Problem 2 O_h magnet

For a ferromagnet with cubic symmetry one can write GL theory with magnetization vector $\mathbf{M} = (M_x, M_y, M_z)$ treated as multi-component order parameter:

$$F[M_x, M_y, M_z] = a(T-T_c)(M_x^2 + M_y^2 + M_z^2) + \frac{1}{2}\beta_1(M_x^2 + M_y^2 + M_z^2)^2 + \frac{1}{2}b(T-T^*)(M_x^4 + M_y^4 + M_z^4)$$

In this functional terms up to fourth power in $\mathbf{M}$, consistent with the cubic symmetry, are kept. Take coefficients $a, b, \beta > 0$ and $T^* < T_c$. This functional supports two solutions, one with $\mathbf{M} \propto < 1, 0, 0 >$ (magnetization along one of the main cubic axes), and another with $\mathbf{M} \propto < 1, 1, 1 >$ (magnetization along cube's diagonal).

Determine the magnetization direction below T_c and below T^*. Find the jump in specific heat at T_c (second order transition) and jump in entropy and latent heat at T^* (first order transition).