Condensed Matter - HW 6 :: Luttinger Liquid

PHSX 545

Problem 1

Compressibility of matter can be described as a response to a scalar potential: by applying pressure at one point (changing chemical potential, or particle density at that point) we should be able to tell how this change affect particle density at some other point. This response is described by the density-density correlation function $\chi(\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2) = \langle \delta\hat{n}(\mathbf{r}_1)\delta\hat{n}(\mathbf{r}_2)\rangle$. In Fourier space this function is (we will derive it in a few weeks)

$$\chi(q) = -\frac{1}{V}\sum_p \frac{n_{p+q/2} - n_{p-q/2}}{\varepsilon_{p+q/2} - \varepsilon_{p-q/2}} = -\int_{-\infty}^{+\infty} \frac{dp}{2\pi\hbar}\frac{n_{p+q/2} - n_{p-q/2}}{\varepsilon_{p+q/2} - \varepsilon_{p-q/2}}$$

where n_p is the occupation number of state with momentum p and energy ε_p.

(a) Show that compressibility, as we defined it earlier, can be directly obtained from density-density correlations

$$\kappa_T = \frac{1}{n}\frac{\partial n}{\partial P} = \frac{1}{n^2}\frac{\partial n}{\partial \mu} = \frac{1}{n^2}\chi(q \to 0)\,,$$

irrespective of the distribution function $n_p(\varepsilon_p)$ (Fermi, Dirac, Boltzmann).

(b) Consider one-dimensional free fermionic gas with $\varepsilon_p = p^2/2m$. At $T = 0$ directly compute the response function $\chi(q)$ and show that it diverges at some wavevector q^*. Find q^* in terms of Fermi momentum p_f and plot $\chi(q)$.

Problem 2

Using the effective Hamiltonian of the charge-spin separated Luttinger model

$$\mathcal{H} = \sum_{q>0}[v_c q\ \hat{a}^\dagger_{c,q}\hat{a}_{c,q} + v_f q\ \hat{a}^\dagger_{s,q}\hat{a}_{s,q}]$$

find the heat capacity of 1D liquid. Hint: think about what values can particle number operators $\hat{n}_{c,s}$ take, and follow the standard route to compute C_V at low temperature.

If one applies Fermi liquid theory to 1D case, what temperature dependence of C_V is expected - is it the same or different?