Condensed Matter - HW1 :: Fermi gas

PHSX 545

Problem 1

Calculate the specific heat of a semiconductor under the assumption $k_B T \ll E_g$ where E_g is the gap between valence and conduction bands. Show that it is given by an ideal gas-like part $(3/2)n(T)k_B$ plus a correction, where $n(T)$ is the number of excitations. Is this correction small or large?

Hint: First, approximate the dispersion of both the conduction and the valence band parabolically, with the two effective masses m_v and m_c. Determine the density of states for the two bands in 3D case. Then, calculate the chemical potential μ from the condition, that the number of electrons in the conduction band $n_e(T)$ must be equal to the number of holes in the valence band $n_h(T)$, under condition $k_B T \ll \mu, E_g - \mu$.

Problem 2

A quasiparticle wave packet is given by a superposition of plane waves

$$\psi_{\mathbf{p}}(\mathbf{r}, t) = \sum_{\mathbf{k}} A_{\mathbf{p}}(\mathbf{k}) e^{i(\mathbf{kr} - \epsilon(\mathbf{k})t)}$$

with Gaussian weight around momentum $\mathbf{p}$:

$$A_{\mathbf{p}}(\mathbf{k}) = C \exp\left(-\frac{(\mathbf{k} - \mathbf{p})^2}{2\Delta k^2}\right)$$

The spread of the wavepacket in momentum space is Δk.

• Find the normalization constant C for 3-dimensional case from $\int d^3\mathbf{r} |\psi_{\mathbf{p}}(\mathbf{r}, t)|^2 = 1$.

• Find the behavior of this wavepacket in real space (how it propagates and its shape), and from it estimate the lifetime of the quasiparticle.

Hint: You may assume that the energy $\epsilon(\mathbf{k})$ does not change drastically on the scale of the wavepacket, and you can use Taylor expansion around $\mathbf{p}$. Take $\partial^2 \epsilon_{\mathbf{p}} / \partial p_i \partial p_j \propto \delta_{ij}$.