Due at noon on May 4, Monday

PHSX 545 Condensed Matter - FINAL EXAM

No collaboration; Open books; Open notes; Please write neatly or type.

Problem 1 2D electrons

Consider low-energy electronic hamiltonian of graphene at half-filling:

$$\mathcal{H} = \sum_{\mathbf{k},s=\pm 1} \varepsilon_{\mathbf{k}s} a^\dagger_{\mathbf{k}s} a_{\mathbf{k}s}$$

with $\mathbf{k} = (k_x, k_y)$, $\varepsilon_{\mathbf{k}s} = s v_f |\mathbf{k}|$, and zero-temperature chemical potential $\mu(0) = 0$.

(a) Sketch the energy dispersion of excitations and determine the low-energy density of states $N(\varepsilon)$;

(b) Show that the chemical potential remains zero for finite T;

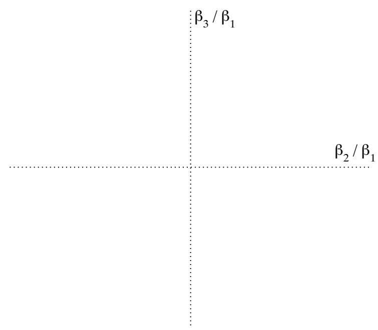
(c) Find the specific heat at low temperature.

Problem 2 Two-component superconductor

A superconductor in tetragonal crystal is described by a two-component order parameter $\boldsymbol{\eta} = (\eta_1, \eta_2)$ that can be treated as a vector in two-dimensional plane. The Ginzburg-Landau functional for this superconductor is given by

$$F[\boldsymbol{\eta}] = \alpha(\boldsymbol{\eta}\cdot\boldsymbol{\eta}^*) + \frac{\beta_1}{2}(\boldsymbol{\eta}\cdot\boldsymbol{\eta}^*)^2 + \frac{\beta_2}{2}|\boldsymbol{\eta}\cdot\boldsymbol{\eta}|^2 + \frac{\beta_3}{2}\left(|\eta_1|^4 + |\eta_2|^4\right)$$

where $\alpha(T) = a(T - T_c)$ and β_i are coefficients (assume $\beta_1 > 0$). Determine the structure of the order parameter, $\eta_{1,2}$, depending on the values of β_3/β_1 and β_2/β_1. Consider phases $\boldsymbol{\eta} \propto (1,0) = (0,1), (1,1), (1,i)$ and on the attached diagram indicate stability region of each.



Problem 3 Magnetic susceptibility

Calculate the spin magnetization in a superconducting state.

(a) Diagonalize the BCS Hamiltonian with spin-singlet isotropic order parameter in the presence of Zeeman magnetic field:

$$\mathcal{H} = \sum_{\mathbf{k},\alpha=\pm 1} (\xi_{\mathbf{k}} - \mu_B H \alpha) a^\dagger_{\mathbf{k}\alpha} a_{\mathbf{k}\alpha} - \sum_{\mathbf{k}} \left(\Delta\, a^\dagger_{\mathbf{k}\uparrow} a^\dagger_{-\mathbf{k}\downarrow} + \Delta^*\, a_{-\mathbf{k}\downarrow} a_{\mathbf{k}\uparrow}\right)$$

(b) Then find the expectation value

$$M = \sum_{\mathbf{k},\alpha=\pm} \langle a^\dagger_{\mathbf{k}\alpha}\, (\mu_B \alpha)\, a_{\mathbf{k}\alpha}\rangle = \mu_B \sum_{\mathbf{k}} \langle a^\dagger_{\mathbf{k}\uparrow}\, a_{\mathbf{k}\uparrow} - a^\dagger_{\mathbf{k}\downarrow}\, a_{\mathbf{k}\downarrow}\rangle = \chi(T) H$$

and write down expression for susceptibility $\chi(T)$.

(c) find limiting behavior of $\chi(T)/\chi_N$ near T_c and in $T \to 0$ limit. χ_N is the normal state magnetic susceptibility of Fermi gas. Discuss your results physically.